



TOBB UNIVERSITY OF ECONOMICS AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

International Conference on Applied Mathematics & Approximation Theory May 17-20, 2012 – Ankara – Turkey

"Celebrating the 60th birthday of Professor George A. Anastassiou"

Conference Chair: Oktay Duman (Turkey)

Organizing Committee: Esra Erkuş-Duman (Turkey)

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Plenary Speakers: George A. Anastassiou (USA)
Dumitru Baleanu (Turkey)
Martin Bohner (USA)
Jerry L. Bona (USA)
Margareta Heilmann (Germany)
Weimin Han (USA)
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ABSTRACTS BOOK

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Description: “Celebrating the 60th birthday of Professor George A. Anastassiou”

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On Neural Network Approximation

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Abstract

Here we present the univariate fractional quantitative approximation of real valued functions on a compact interval by quasi-interpolation sigmoidal and hyperbolic tangent neural network operators. These approximations are derived by establishing Jackson type inequalities involving the moduli of continuity of the right and left Caputo fractional derivatives of the engaged function. The approximations are pointwise and with respect to the uniform norm. The related feed-forward neural networks are with one hidden layer. Our fractional approximation results of higher order converge better than the ordinary ones. We further deal with the determination of the fractional rate of convergence to the unit of other neural network operators such as Cardaliaguet-Euvrard and “squashing” operators. We discuss also the multivariate approximation by multivariate analogs of the above mentioned neural network operators. When operators are normalized results become more elegant. We finish with related Voronovskaya asymptotic expansions.

Fractional Inequalities Involving Convexity

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Abstract

We present a series of general inequalities for integral operators involving convexity. As applications of these we derive inequalities involving fractional Riemann-Liouville integrals and their generalizations, then for three kinds of basic fractional derivatives and their radial versions in the multivariate case. Our inequalities engage products of functions and in their right hand side separate. So we get Hardy and Poincare type fractional inequalities.

Open Problems in the Area of Fractional Calculus and its Applications

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Abstract

Fractional calculus which is as old as the classical calculus has become a candidate in solving problems of complex systems which appears in various branches of science and engineering [1-4]. In this talk we discuss some open problems of this type of calculus in the area of mathematics, physics and bio-engineering. Several illustrative examples from each branch of investigated fields will be given.

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The Spectrum of a q -Difference Operator

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Abstract

This talk reports on the three papers [1, 2, 3] related to the spectrum of a certain q -difference operator. For a number q bigger than one, we consider a q -difference version of a second-order singular differential operator which depends on a real parameter. We give three exact parameter intervals in which the operator is semibounded from above, not semibounded, and semibounded from below, respectively. We also provide two exact parameter sets in which the operator is symmetric and self adjoint, respectively. Our model exhibits a more complex behaviour than in the classical continuous case but reduces to it when q approaches one. For values of the parameter for which the difference operator is self adjoint, we show that the spectrum of the operator is discrete and simple. When q approaches 1, the spectrum fills the whole positive or negative semiaxis.

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Theory and Application of Water Wave Models

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Abstract

The discussion begins with a few historical remarks. Once the context is set, recent theoretical work on water wave models will be presented. These results are then used to investigate questions arising in laboratory and field situations. As time permits, these will include topics such as tsunami propagation, the generation of rogue waves and sand bar formation and stability.

On a Family of Models in X -ray Dark-Field Tomography

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Abstract

X -ray mammography is currently the most prevalent imaging modality for screening and diagnosis of breast cancers. However, its success is limited by the poor contrast between healthy and diseased tissues in the mammogram. A potentially prominent imaging modality is based on the significant difference of x -ray scattering behaviors between tumor and normal tissues. Driven by major practical needs for better x -ray imaging, exploration into contrast mechanisms other than attenuation has been active for decades, e.g., in terms of scattering, which is also known as dark-field tomography. In this talk, a theoretical study is provided for the x -ray dark-field tomography (XDT) assuming the spectral x -ray detection technology.

The radiative transfer equation (RTE) is usually employed to describe the light propagation within biological medium. It is challenging to solve RTE numerically due to its integro-differential form and high dimension. For highly forward-peaked media, it is even more difficult to solve RTE since accurate numerical solutions require a high resolution of the direction variable, leading to prohibitively large amount of computations. For this reason, various approximations of RTE have been proposed in the literature. For XDT, a family of differential approximations of the RTE is employed to describe the light propagation for highly forward-peaked medium with small but sufficient amount of large-angle scattering. The forward and inverse parameter problems are studied theoretically and approximated numerically.

New Results for Genuine Szász-Mirakjan-Durrmeyer Operators

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Abstract

We consider a variant of Szász-Mirakjan-Durrmeyer operators preserving linear functions and can therefore be named as genuine Szász-Mirakjan-Durrmeyer operators in the same meaning as genuine Bernstein-Durrmeyer and genuine Baskakov-Durrmeyer operators.

For a function $f \in C[0, \infty)$ satisfying an exponential growth condition, i.e., $|f(t)| \leq Me^{\alpha t}$, $t \in [0, \infty)$, for a constant $M > 0$ and $\alpha > 0$, the operators $\tilde{S}_n$, $n > \alpha$, are defined by

$$\tilde{S}_n(f, x) = e^{-nx}f(0) + n \sum_{k=1}^{\infty} s_{n,k}(x) \int_0^{\infty} s_{n,k-1}(t)f(t)dt,$$

where

$$s_{n,k}(x) = \frac{(nx)^k}{k!} e^{-nx}, \quad k \in \mathbb{N}_0, \quad x \in [0, \infty).$$

Up to our current knowledge these operators were first defined by R. S. Phillips and therefore often are called Phillips operators. Among others the operators were studied by C. P. May [4] and more recently by Z. Finta [3] and Z. Finta and V. Gupta [2] who proved a strong converse result of type B in the terminology of Z. Ditzian and K. G. Ivanov [1].

As main results we will present commutativity properties of the operators as well as the commutativity with an appropriate differential operator and a strong converse result of type A in terms of a K-functional with an explicit constant. Together with a direct theorem this leads to an equivalence result for the error of approximation and the K-functional and the corresponding Ditzian-Totik modulus of smoothness, respectively.

Keywords: Genuine Szász-Mirakjan operators, Phillips operators, commutativity results, strong converse result.

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Weak Filter Convergence for Unbounded Sequences

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Abstract

[This talk is based on a joint research with V.Kadets and A.Leonov].

Let H be an infinite-dimensional separable Hilbert space. It is well known that the properties of sequences that are filter convergent in the weak topology of H differ significantly from the properties of the ordinary weakly convergent sequences. In particular a weakly convergent sequence must be bounded but, say, a weakly statistically convergent sequence $h_n \in H$ can tend to infinity in norm. This effect induces the following natural question:

If a sequence has a weak limit with respect to a given filter $\mathcal{F}$, how quick can the norms of the elements in the sequence tend to infinity?

Of course the answer depends on filter. In this talk we mostly concentrate our efforts on the statistical convergence filter and on its direct generalization-the Erdős-Ulam filters. Some general results are also given.

Open Problems in Semi-linear Uniform Spaces

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Abstract

Semi-linear uniform space is a new space defined by Tallafha, A and Khalil, R in [7], the authors studied some cases of best approximation in such spaces, and gave some open problems in uniform spaces. Besides they defined a set valued map ρ on $X \times X$ and asked two questions about the properties of ρ . In 2011, Tallafha [8] defined another set valued map δ on $X \times X$ and give more properties of semi-linear uniform spaces using the maps ρ, δ and he answered two of these question. The purpose of this talk is to introduce semi-linear uniform space. Also we shall gave a lot of open questions concerning this new spaces.

Keywords: Best approximation, Uniform spaces, Semi-linear.

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On Generalized k -Primary Rings

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Abstract

The present paper introduces and studies certain types of rings and ideals such as generalized k -primary rings (resp. generalized k -primary ideals), principally generalized k -primary rings (resp. principally generalized k -primary ideals) and completely generalized k -primary rings (resp. completely generalized k -primary ideals). Some properties of them are obtained and some characterizations of each type are given.

Keywords: gkp -rings, $pgkp$ -rings, $cgkp$ -rings.

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Fractional Schrödinger operators in one-dimension

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Abstract

On the separable infinite dimensional Hilbert space $\mathcal{H} = L^2(\mathbb{R}, dx)$ we consider the Schrödinger operator $\hat{H}_g = K_\alpha \hat{\mathcal{P}}^\alpha + g\hat{V}$ where $\hat{\mathcal{P}}^\alpha$ is the space fractional Weyl operator, g is a small coupling constant and the potential $V \in L^{\gamma+1/\alpha}(\mathbb{R})$, $\alpha \in (1, 2]$. We study two problems:

1. The existence and uniqueness of bound states and especially the calculation of a sharp estimate for the lowest eigenvalue of $\hat{H}_g$ [1].
2. The Lieb-Thirring inequality for the sum of powers of eigenvalues.

For the first problem, necessary and sufficient conditions for there to be a ground state for g are given and then applied to study two instructive examples.

Keywords: Fractional Schrödinger operator, Birman-Schwinger representation, Asymptotic behaviour of ground state, Lieb-Thirring inequality.

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A Hybrid Method for Inverse Scattering Problem for a Dielectric

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Abstract

The inverse problem under consideration is to reconstruct the shape of a homogeneous dielectric infinite cylinder from the far field pattern for scattering of a time-harmonic E-polarized electromagnetic plane wave. We propose an inverse algorithm that extends the approach suggested by Kress, Serranho [1,2,3] for the case of the inverse problem for a perfectly conducting scatterer to the case of penetrable scatter. It is based on a system of nonlinear boundary integral equations associated with a single-layer potential approach to solve the forward scattering problem. We present the mathematical foundations of the method and exhibit its feasibility by numerical examples.

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Solving Second Order Discrete Sturm-Liouville BVP Using Matrix Pencils

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Abstract

This paper deals with discrete second order Sturm-Liouville Boundary Value Problems (DSLBP) where the parameter appears nonlinearly in the boundary conditions. We focus on analyzing the DSLBP with cubic nonlinearity in the boundary condition. The problem is described by a matrix equation with nonlinear variables. By applying the matrix pencil techniques, a DSLBP can be viewed as a generalized eigenvalue problem with respect to its coefficient matrix. Under certain conditions, it can be further reduced to a regular eigenvalue problem so that many existing computational tools can be applied to solve the problem. The main results of the paper provide the reduction procedure and methods to identify those cubic DSLBPs which can be reduced to regular eigenvalue problems. We also investigate the structure of the coefficient matrix of a DSLBP and its effect on the reality of the corresponding eigenvalues.

Keywords: Sturm-Liouville Boundary Value Problem, Eigenvalue, Matrix Pencil.

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On Univalence of a General Integral Operator

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Abstract

Problem statement: We introduce and study a general integral operator defined on the class of normalized analytic functions in the open unit disk. This operator is motivated by many researchers. With this operator univalence conditions for the normalized analytic function in the open unit disk are obtained. In deed, the preserving properties of this class are studied, when the integral operator is applied and we present a few conditions of univalency for our integral operator. The operator is essential to obtain univalence of a certain general integral operator. **Approach:** In this paper we discuss some extensions of univalent conditions for an integral operator defined by our generalized differential operator. Several other results are also considered. We will prove in this paper the univalent conditions for this integral operator on the class of normalized analytic functions when we make some restrictions about the functions from definitions. **Results:** Having the integral operator, some interesting properties of this class of functions will be obtained. Relevant connections of the results, shall be presented in the paper. In fact, various other known results are also pointed out. We also find some interesting corollaries on the class of normalized analytic functions in the open unit disk. **Conclusion:** Therefore, many interesting results could be obtained and we also derive some interesting properties of these classes. We conclude this study with some suggestions for future research, one direction is to study other classes of analytic functions involving our integral operator on the class of normalized analytic functions in the open unit disk.

Keywords: Analytic functions; Univalent functions; Derivative operator; Hadamard product.

Boundedness of pseudo-differential operator involving fractional Fourier transform

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Abstract

Pseudo-differential operator A_a associated with fractional Fourier transform involving the symbol $a(x, \xi)$ is defined. An integral representation of pseudo-differential operator A_a and boundedness of the composition of operators Δ_x^r and A_a are defined. An integral operator $A_{a,\alpha}$ is defined and studied its boundedness property.

Keywords: Pseudo-differential operator, fractional Fourier transform, Sobolev space.

On exact values of monotonic random walks characteristics on lattices

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Abstract

Traffic flow models, that are based on cellular automata and monotonic random walks, are investigated in [1, 2, 3, 4] et al. We consider here a monotonic random walk of particles on a one dimensional lattice. Jumps of particles can be occurred in discrete times and are realized with probability depending on the type of the particle and coordinate of the cell occupied by the particle, i.e. this particle comes to the cell adjacent to the particle that moves ahead (model with maximum transitions). Let us describe a stochastic model of particles movement on a closed sequence of cells. Let the number of cells be equal to n . The particles move at the discrete times $1, 2, \dots$. Each cell is occupied by no more than one particle. There are k types of particles. The number of particles is equal to m , $1 < m < n$. There are m_s particles of the type s . Let the i -th particle be a particle of the $s(i)$ -th type. Suppose that a particle of the s -th type occupies the i -th cell and there are d empty cells in front of this particle. Then the particle passes the maximum number of cells with the probability p_{is} , $0 < p_{is} < 1$, i.e., it comes to the cell followed the particle ahead at the moment of moving. Suppose that the numbers n and m are coprime. Let q be the flow intensity, i.e., the average number of particles passing through a cell per a time unit. Denote $r = m/n$, $r_s = m_s/n$. The value r is the particles flow density. The value r_s is the flow component composed by particles of the s -th type. We proved that the following formula is true

$$q = nr(1 - r) \left(\sum_{i=1}^n \sum_{s=1}^k \frac{r_s}{p_{is}} \right)^{-1}.$$

Thus formula for the flow intensity has been found.

Keywords: Monotonic random walk, traffic mathematical models.

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Three Algorithms for the Assembly Flowshop Scheduling Problem

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Abstract

The two stage assembly flowshop scheduling problem has a lot of applications, and hence, it has been addressed in the scheduling literature with respect to different performance measures such as makespan, total completion time, or maximum lateness. The performance measure of total tardiness is also important in situations where there is a penalty to complete a job beyond its due date, and the penalty increases as the gap between the job's due date and its completion time increases. Such costs may also be penalty costs in contracts, loss of goodwill, and damaged reputation. To the best of our knowledge, the problem with the objective of minimizing total tardiness has not been addressed so far, and hence, it is addressed in this paper. First mathematical formulation of the problem is presented, and next three algorithms are developed for the problem. The developed algorithms are; an insertion algorithm, a genetic algorithm, and a hybrid of the insertion and genetic algorithms. Conducted computational experiments reveal that the overall relative errors of the insertion, genetic, and hybrid algorithms are 0.516, 6.055, and 0.763, respectively. This clearly indicates that the hybrid algorithm significantly outperforms the genetic algorithm while the best performing algorithm is the insertion algorithm. Moreover, the performance of the insertion algorithm remains as the best regardless of the tightness of due dates of the jobs. This further indicates the strength of the insertion algorithm.

Keywords: Mathematical formulation, algorithm, scheduling, total tardiness.

Alzer Inequality for Hilbert Spaces Operators

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Abstract

In this paper, we give an extension of Alzer inequality for Hilbert space operators as follows:

Let $A_1, \dots, A_n$ be n selfadjoint operators on an Hilbert space $\mathcal{H}$ such that $0 < A_j \leq \frac{1}{2}I$, where I is identity operator on $\mathcal{H}$ [1, 2]. Also, let $\mathcal{A}_n := \mathcal{A}_n(A_1, \dots, A_n)$ and $\mathcal{G}_n := \mathcal{G}_n(A_1, \dots, A_n)$ be arithmetic and geometric means of $A_1, \dots, A_n$ [7], and $\mathcal{A}'_n := \mathcal{A}_n(A'_1, \dots, A'_n)$ and $\mathcal{G}'_n := \mathcal{G}_n(A'_1, \dots, A'_n)$ be arithmetic and geometric means of $A'_1, \dots, A'_n$ where $A'_j := I - A_j$ ($j = 1, \dots, n$), respectively. Then we show that

$$\mathcal{A}'_n - \mathcal{G}'_n \leq \mathcal{A}_n - \mathcal{G}_n.$$

Keywords: operator concavity, selfadjoint operator, arithmetic mean, geometric mean, harmonic mean.

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Edgeworth Approximation for Some Distributions in Business And its Application in the Black-Scholes Option Pricing Model

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Abstract

In this paper, we study the Edgeworth second order expansion and its asymptotic behavior. The form of the series depends asymptotically on the skewness and kurtosis of the underlying distribution. We also defined its validity region where the series behaves as a positive definite and unimodal probability density function. We also assume that the underlying population has finite first four moments. We illustrate with examples these findings on four types of continuous distributions that have significant applications in business, economics and finance; lognormal, pareto, logistic and chi-squared distributions. Lognormal distribution is mainly used to describe the calculated returns over a specific period of time. While, Logistic can be used in logistic management or supply chain management. Pareto distribution is useful in the area of continuous improvement and value engineering: A systematic method for analyzing a design, product, process, project or construction to improve performance and quality, while reducing the associated costs. Its known that by using Pareto, business processes can be greatly improved by reducing customer complaints and organizational problems. In finance it is useful in improving financial planning and personal time management. In the end, we considered the option pricing problem in the Black-Scholes model, to estimate the fair price using Edgeworth second order approximation.

Keywords: Black-Scholes model, Chi-Squared, Edgeworth expansion, Lognormal, Logistic, Pareto.

Approximation of small probabilities of the sums of random number of summands

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Abstract

By

$$Z_N = \sum_{j=1}^N a_j X_j, \quad Z_0 = 0 \quad (1)$$

we denote a sum of a random number of summands. Here $\{X, X_j, j \geq 1\}$ is a family of independent identically distributed random variables (r.v.s.) with variance $\mathbf{D}X = \sigma^2 > 0$ and mean $\mathbf{E}X = \mu < \infty$. In this scheme of summation we have to consider two cases: $\mu \neq 0$ and $\mu = 0$. In addition, it is assumed that $0 \leq a_j < \infty$, and a non-negative integer-valued random variable (r.v.) N is independent of X_j . Random sums (r.s.) (1) appears as models in many applied problems, for instance, in stochastic processes, queue theory, insurance, finance mathematics and is essential in other fields too.

We consider the standard normal approximation, large deviation theorems in the Cramer and power Linnik zones, asymptotic expansions for the distribution density for the sum $\tilde{Z}_N = (Z_N - \mathbf{E}Z_N)/(\mathbf{D}Z_N)^{1/2}$, exponential inequalities for the tail probability $\mathbf{P}(\tilde{Z}_N \geq x)$, under assumptions for the r.v.'s $X, T_{N,1}, T_{N,2}$ moments and cumulants, respectively. Here $T_{N,r} = \sum_{j=1}^N a_j^r$, $r \in \mathbb{N}_0$. We restrict our attention to the cumulant method, offered by V. Statulevičius (see [1]). In addition, without it the characteristic method is used.

The results are obtained in two cases: $\mu = 0, \mu \neq 0$. Moreover, the instances of large deviations: a discount version, the case $a_j \equiv 1$, besides, examples when N is non-negative, obey Poisson, binomial, Bernoulli, negative binomial, geometric laws are considered. Note that large deviation equalities for r.s. (1) in the Cramer and power Linnik zones, in the case where $\mu \neq 0$ are presented in our paper [1], and the discount version of large deviations can be found in [1]. For instance, large deviation theorems in the Cramer zone for r.s. (1), in the case $a_j \equiv 1$ have been proved in the paper [3].

Keywords: cumulant method, large deviations, random sums.

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New Approach for Multidimensional Scaling with Categorical Data

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Abstract

Multidimensional scaling is the problem of representing n objects geometrically by n points, so that the interpoint distances correspond in some sense to experimental dissimilarities between objects. In this paper we consider a parametric family of multivariate multinomial distributions. We observe realizations w of W with

$$w = (h_{11}, \dots, h_{k1}, h_{12}, \dots, h_{kL}).$$

Here all frequencies h_{il} are nonnegative, $(h_{1l}, \dots, h_{kl})$ is a realization of W_l with

$$\sum_{i=1}^k h_{il} = \tilde{n}_l, \quad P(h_{1l}, \dots, h_{kl}) = \frac{\tilde{n}_l!}{h_{1l}! \cdot \dots \cdot h_{kl}!} p_1(\mu, t_l)^{h_{1l}} \cdot \dots \cdot p_k(\mu, t_l)^{h_{kl}}.$$

Here A categorical data is considered, for these data we get a new type of stress function. The computational results show a good approach with this type of data.

Keywords: Multidimensional Scaling, Stress Function, Categorical Data.

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Difference scheme of higher accuracy order for solution of the Dirichlet's problem¹

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Abstract

In present work for numerical solving Dirichlet's problem for Laplace's equation, there is applied difference scheme of higher accuracy order, allowing getting estimate of the error of order $O(h^4)$, only known data take part in this estimate.

Representation of difference problem (five-point scheme) solution for Laplace's equation on rectangle with aid of discrete analog of Fourier method, suggested and grounded in [1], is the main tool for creating new economical methods for finding its solutions both on whole net domain [2], and on net segments [3].

Let us denote through Π a rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, b)$, $(0, b)$, where b – rational number. Let Γ – boundary of this rectangle. Let, further $\Gamma_{1h} = \{(x, y) : x = ih, i = 1, \dots, n, y = 0\}$.

Consider Dirichlet's problem

$$\begin{cases} \Delta u = 0 \text{ on } \Pi, \\ u|_{\Gamma} = f. \end{cases} \quad (1)$$

where f – defined on Γ and has fifth derivative on each side of Γ .

Through $\Delta_h u$ denote Laplace's nine-point difference operator. Through $Q(x, y)$ denote special polynomial of fourth degree. If we replace boundary function f with $\varphi_h = f - Q$, then we receive new problem, for which the estimate error is the same as for our problem. Through φ_h denote assigned function on Γ_{1h} :

$$\varphi_h = \begin{cases} 0, & \text{on vertexes } \Pi, \\ f - Q, & y = 0. \end{cases}$$

An error of the method is estimated as followed:

$$|u - u_h| \leq ch^4.$$

Keywords: Difference scheme, equation, solution.

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Fractional Calculus Models in DNA

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Abstract

In this paper we introduce the fractional calculus models in DNA systems

Keywords: Fractional Calculus, DNA Systems, Fourier Transform.

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Monomorphic structures and homogeneous groups

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Abstract

Given a set E and an integer m , a relation of arity m and basis E is an application of the m -tuple $(x_1, \dots, x_m)$ set in the set of two values $+$ and $-$. Digraphs can be considered as binary relations (ie of arity 2). Two relations R and R' of arity m and with respective basis E and E' are isomorphic if there exists a bijection f from E to E' such that for any m -tuple $(x_1, \dots, x_m)$ of elements of E , we have $R(x_1, \dots, x_m) = R'(f(x_1), \dots, f(x_m))$. A relation of arity m is p -monomorphic if all its restrictions on p elements are isomorphic.

We present some results on the conjecture of Pouzet [1] concerning monomorphic relations and homogeneous groups. We expound as well some generalisation of monomorphy.

Keywords: Binary relations, Digraph, Monomorphy.

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A Deterministic Inventory Model of Deteriorating Items with Stock and Time Dependent Demand Rate

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Abstract

In formulating inventory models, two facts of the problem have been of growing interest, one being the deterioration of items, the other being the variation in the demand rate. Time-varying demand patterns are usually used to reflect sales in different phases of the product life cycle in the market. The effect of deterioration of physical goods cannot be disregarded in many inventory systems. Deterioration is defined as decay, damage and spoilage. Food items, photographic films, drugs, pharmaceuticals, chemicals, electronic components and radioactive substances are some. A deterministic inventory model for deteriorating item with inversely time dependent of two parameter Weibull distributions to represent the deterioration rate has been studied in this paper. Time dependent and stock dependent demand rate separately has been studied by numerous authors while in this paper considering simultaneously both stock dependent and time dependent demand rate has been studied. The present model has been solved analytically to minimize the cost. A numerical example has been carried out to illustrate the solution procedure.

Keywords: Inventory, Deterioration, Weibull distribution.

Approximation Formulas for the Moments of Gaussian Random Walk with a Reflecting Barrier

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Abstract

In this study, a semi-Markovian random walk process with a generalized reflecting barrier is constructed mathematically. Under some weak conditions, the ergodicity of the process is proved and exact form of the first four moments of the ergodic distribution is obtained. After, the asymptotic expansions of the moments are established. The coefficients of the asymptotic expansions are expressed by means of numerical characteristics of a residual waiting time. Finally, the accuracy of the approximation formulas compared to exact expressions for the ergodic moments is tested by Monte Carlo simulation method.

Keywords: Gaussian random walk, Reflecting barrier, Ergodic moments, Residual waiting time, Approximation formula.

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On the g -Jacobi Matrix Functions

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Abstract

This paper attempts to present a matrix extension of the generalized Jacobi (g -Jacobi) function which is a solution of fractional Jacobi differential equation. Various properties of this function are obtained. Furthermore, the fractional hypergeometric matrix function is introduced as a solution of the matrix generalization of the fractional Gauss differential equation. Finally, some special cases are given.

Keywords: g -Jacobi function, fractional derivative, matrix theory, hypergeometric function, Gauss differential equation.

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Nonlinear wavelet regression function estimator for censored data under α -mixing condition

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Abstract

In this paper we introduce a new nonlinear wavelet-based estimator of the regression function in the right censorship model. An asymptotic expression for the mean integrated squared error (MISE) of the estimator is obtained to both continuous and discontinuous curves. It is assumed that the lifetime observations from a stationary α -mixing sequence.

Keywords: Censored data, Mean integrated squared error, Nonlinear wavelet-based estimator, Nonparametric regression, Strong mixing condition.

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Existence results for a fractional boundary value problem

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Abstract

We establish sufficient conditions for the existence and uniqueness of solutions for boundary value problem for fractional differential equations. For this we apply some fixed point theorems.

Keywords: Fractional Caputo derivative, Banach Contraction principle, Leray Schauder nonlinear alternative.

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A generalization of some orthogonal polynomials

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Abstract

In this paper we show how the action of the operator $\pi_{e_1 e_2}$ of the series $\sum_{j=0}^{\infty} S_j(A) z^j$ allows us to obtain a generalization of fibonacci numbers and certain results of Foata and Ramanujan, and other result on Tchebcheff polynomials of first and second case.

Keywords: Chebyshev polynomials, symmetric functions, Fibonacci numbers.

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Robust preconditioners for the high-contrast Stokes problem

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Abstract

We study the Stokes equation with high-contrast viscosity coefficient and this regime corresponds to a small Reynolds number regime because viscosity is inversely proportional to the Reynolds number. Numerical solution to the Stokes flow problems especially with high-contrast variations in viscosity is critically needed in the computational geodynamics community. One of the main applications of the high-contrast Stokes equation is the study of earth's mantle dynamics.

The high-contrast coefficient creates small eigenvalues which prohibits the utilization of traditional iterative solvers. In order to overcome solver difficulties, we construct a preconditioner that is robust with respect to contrast size and mesh size simultaneously based on the preconditioner proposed by Aksoylu et al. [1]. One of the strengths of our proposed preconditioner is rigorous justification. The proposed preconditioner was originally designed for the high-contrast diffusion equation under finite element discretization [1]. Rigorous justification has been obtained through the usage of singular perturbation analysis (SPA). Aksoylu and Yeter [2] extended the proposed preconditioner from finite element discretization to cell-centered finite volume discretization. Hence, we have shown that the same preconditioner could be used for different discretizations with minimal modification. Furthermore, Aksoylu and Yeter [3] applied the same family of preconditioners to high-contrast biharmonic plate equation. Such dramatic extensions rely on the generality of the employed SPA. Therefore, we have accomplished a desirable preconditioning design goal by using the same family of preconditioners to solve the elliptic family of PDEs with varying discretizations. In this article, we aim to bring the same rigorous preconditioning technology to vector valued problems such as the Stokes equation.

Keywords: Stokes equation, Stokes flow, high-contrast, high-contrast viscosity, discontinuous coefficients, Uzawa solver, saddle point problem, singular perturbation analysis, Schur complement, heterogeneity, viscosity.

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Dynamical Behavior of a Ratio Dependent Predator-Prey System with Distributed Delay

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Abstract

In this study, we consider a predator-prey system with distributed time delay where the predator dynamics is logistic with the carrying capacity proportional to prey population. In [1] and [2], we studied the impact of the discrete time delay on the stability of the model, however in this study, we investigate the effect of the distributed delay for the same model. By choosing the delay time τ as a bifurcation parameter, we show that Hopf bifurcation can occur as the delay time τ passes some critical values. Using normal form theory and central manifold argument, we establish the direction and the stability of Hopf bifurcation. Some numerical simulations for justifying the theoretical analysis are also presented.

Keywords: Predator-prey system, distributed delay, hopf bifurcation, stability.

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On Supremum, Infimum, Maximum Gain and Maximum Loss of Brownian Motion with drift and of Fractional Brownian Motion

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Abstract

In finance one of the primary issues is managing risk. Related to this issue and maybe for hedging, investors are naturally interested in the expected values of supremum, infimum, maximum gain and maximum loss of risky assets and the relations between them. Price of a risky asset, stock, can be modeled using Brownian motion and fractional Brownian motion. In this study, we first present the marginal and joint distributions of supremum, infimum, maximum gain and maximum loss of Brownian motion with drift, 0. As an extension of this work, we provide calculations of the expectations and correlation between them for Brownian motion with drift. We give results related to these distributions over various time horizons. We also present numerical studies of Brownian motion with drift and we collect some conjectures on the relation between maximum gain and maximum loss of stock prices. We introduce some bounds on the expected values and distributions of supremum and of maximum loss of fractional Brownian motion. We present large deviation results on maximum loss of fractional Brownian motion, which is also an alternative model of risky asset to Brownian motion.

Keywords: Brownian motion with drift, Markov Property, Correlation Coefficient, Fractional Brownian Motion, Self Similarity Property, Hitting Time, Laplace Transform, Hurst Parameter, Markov's Inequality, Large Deviations

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Statistical Convergence on Time Scales

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Abstract

In this talk, we introduce the statistical convergence of Δ -measurable real-valued functions defined on time scales. This convergence method combines the definitions given by Fast [1] for discrete case and Móricz [2] for continuous case. We obtain some characterizations on statistical convergence and investigate some fundamental properties. Various applications are also presented.

Keywords: Time scales, Δ -measurable function, statistical convergence.

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Matlab Codes to Solves the Static Bending of a Linear Elastic Beam

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Abstract

In this paper, we describe a Matlab Codes (program) to solves the static bending of a linear elastic beam. First, we search the exact solution for this problem. In the next step, we solves a linear elastic 2D beam problem (plane stress or strain) with several element types.

Keywords: Matlab Program, Exact Solution, Finite Difference Method, Approximation Solution.

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Nonstandard Finite Difference Schemes for Fuzzy Differential Equations

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Abstract

In this paper, a method for numerical solutions of fuzzy first order initial value problem is presented. We construct and develop nonstandard scheme for fuzzy differential equations. Examples are given, including nonlinear fuzzy first order differential equations.

Keywords: Fuzzy differential equations, nonstandard finite difference schemes, fuzzy numbers, numerical solutions.

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Fixed Point Theorems for Generalized Contractions in Ordered Uniform Space

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Abstract

In this work, we use the order relation on uniform spaces which is defined by [1] so we present some fixed point results for monotone operators in ordered uniform spaces using a weak generalized contraction-type assumption.

Keywords: Fixed points, Ordered uniform spaces, Generalized contractions.

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Extenssion of Karmarkar's algorithm for solving an optimization problem

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Abstract

In this paper, we propose an algorithm of a interior point methods to solve a linear complementarity problem (*LCP*). The study is based on the transformation of a linear complementarity problem (*LCP*) into a convex quadratic problem, then we use the linearization approach for obtain the simplified problem of Karmarkar. Theoretical results deduct of those established later and the numerical tests confirm that the algorithm is robust.

Keywords: Quadratic programming, Convex non linear programming, Interior point methods.

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Oscillatory Behaviour of Solutions of Fourth Order Delay and Advanced Dynamic Equations

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Abstract

A time scale is a nonempty closed subset of real numbers. The study of dynamic equations on time scales is a unification and extension of the theories of continuous and discrete analysis. It is introduced by Stefan Hilger [11] in 1988. In this talk, some oscillatory criteria for fourth order dynamic equations on time scales are given. Our main results in this paper are even new in discrete cases.

Keywords: Oscillation, fourth order, dynamic equations, time scales.

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On Partial Metric Spaces and Some Related Fixed Point Theorems

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Abstract

In 1992, Matthews [1, 2] introduced the notion of a partial metric space which is a generalization of usual metric spaces in which $d(x, x)$ are no longer necessarily zero. He also proved the analog of Banach contraction principle in the context of partial metric spaces. In this talk, some recent fixed point theorems in the class of partial metric spaces are discussed (see e.g. [3]-[7]).

Keywords: Partial Metric Spaces, Fixed Point, Contractions.

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Asymptotic Distribution of Vector Variance Standardized Variables without Duplications

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Abstract

In recent years, the use of vector variance standardized variables as a measure of testing equality of correlation matrices has received much attention in wide range of statistics. This paper deals with a more economic measure of testing equality of correlation matrices, defined as vector variance standardized variables minus all duplication elements. For high dimensional data, this will increase the computational efficiency almost 50 % compared to the original vector variance standardized variables. Its sampling distribution will be investigated to make its applications possible.

Keywords: Asymptotic distribution, correlation matrix, likelihood ratio test, vector variance standardized variables, vector variance.

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Harmonic - Geometric - Arithmetic Mean Inequality of Several Positive Operators

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Abstract

In this paper we define harmonic, geometric and arithmetic means of several positive operators in $B(\mathcal{H})$ and we show the inequality between of its. Also we prove the generalized geometric-arithmetic mean inequality as follows:

We take $\mathbf{a}_2(A, B) = A \nabla_p B$, $\mathbf{g}_2(A, B) = A \sharp_p B$ and $\mathbf{h}_2(A, B) = A !_p B$. With the above notations, if $\mathbf{B} = (B_1, \dots, B_n)$ be a sequence of positive operators and $\lambda = (\lambda_1, \dots, \lambda_n)$ be a sequence of positive number so that $\lambda_1 + \dots + \lambda_n = 1$. We define arithmetic, geometric and harmonic, mean of several operators as follows:

$$\mathbf{a}_n(\mathbf{B}, \lambda) = \mathbf{a}_2(B_1, \mathbf{a}_{n-1}(\mathbf{B}', \lambda'), \lambda_1)$$

$$\mathbf{g}_n(\mathbf{B}, \lambda) = \mathbf{g}_2(B_1, \mathbf{g}_{n-1}(\mathbf{B}', \lambda'), \lambda_1)$$

$$\mathbf{h}_n(\mathbf{B}, \lambda) = \mathbf{h}_2(B_1, \mathbf{h}_{n-1}(\mathbf{B}', \lambda'), \lambda_1)$$

where $\mathbf{B}' = (B_2, \dots, B_n)$ and $\lambda' = (\lambda'_1, \dots, \lambda'_{n-1})$, that $\lambda'_1 + \dots + \lambda'_{n-1} = 1$. Then we have

$$\mathbf{h}_n(\mathbf{B}, \lambda) \leq \mathbf{g}_n(\mathbf{B}, \lambda) \leq \mathbf{a}_n(\mathbf{B}, \lambda).$$

Keywords: positive operator, geometric mean, arithmetic mean, harmonic mean.

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A Modified Adomian Approach Applied to Nonlinear Fredholm Integral Equations

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Abstract

In this paper, we introduce the linearization method and the modified Adomian method applied to non linear Fredholm integral equations. To assess the applicability, simplicity and the accuracy of the modified Adomian technique, we applied the both methods on selected non-linear Fredholm integral equations. This study showed the applicability, simplicity, accuracy and the fast speed of convergent of the modified Adomian method, comparing with the linearization method, even when the accuracy of the linearization method improved by employing variable steps size.

Keywords: Adomian method, linearization method, non-linear integral equations.

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Pairwise Likelihood Procedure To Estimate A Shift Parameter

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Abstract

This study is about estimating the shift parameter in the two-sample location problem. The proposed procedure uses pairwise differences of the random samples to find a distribution function of the differences. Hence, using the distribution function of the pairwise differences, one can find a likelihood function with respect to the shift parameter. The shift parameter can be estimated either explicitly or with iteration such as Newton's one step estimator by solving the likelihood function. Moreover, it will be shown that the proposed testing procedure is equivalent to Rao's score type test. Also, the theory of the proposed procedure is similar to the regular maximum likelihood theorems. An asymptotic level hypothesis test and confidence interval will be investigated for the proposed procedure. The study ends with a bootstrap simulation study to show the efficiency of the shift estimator.

Keywords: Location Problem; Pairwise Likelihood; Likelihood Ratio; Shift Parameter

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Inverse Spectral Problems for Complex Jacobi Matrices

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Abstract

An $N \times N$ complex Jacobi matrix is a matrix of the form

$$J = \begin{bmatrix} b_0 & a_0 & 0 & \cdots & 0 & 0 & 0 \\ a_0 & b_1 & a_1 & \cdots & 0 & 0 & 0 \\ 0 & a_1 & b_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{N-3} & a_{N-3} & 0 \\ 0 & 0 & 0 & \cdots & a_{N-3} & b_{N-2} & a_{N-2} \\ 0 & 0 & 0 & \cdots & 0 & a_{N-2} & b_{N-1} \end{bmatrix}, \quad (1)$$

where for each n , a_n and b_n are arbitrary complex numbers such that a_n is different from zero:

$$a_n, b_n \in \mathbb{C}, \quad a_n \neq 0. \quad (2)$$

The general inverse problem is to reconstruct the matrix given some of its spectral characteristics (spectral data) [1, 2, 3].

In this talk, we introduce the concept of spectral data for matrices (1) with entries satisfying (2) and present a solution of the inverse problem of recovering the matrix from its spectral data. We give an application to the solving of finite complex Toda lattices by the method of inverse spectral problem.

Keywords: Jacobi matrix, spectral data, inverse problem.

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State Dependent Sweeping Process with Perturbation

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Abstract

Several extensions of the sweeping process in diverse ways obtained; see for example [1, 2]. In this work, we prove the existence of solutions for the following state dependent sweeping process with perturbation

$$(\mathcal{P}) \begin{cases} -\dot{u}(t) \in N_{C(t,u(t))}(u(t)) + F(t, u(t)) & \text{a.e on } [0, T] \\ u(0) = u_0 \in C(0, u_0), \end{cases}$$

where $N_{C(t,u(t))}(\cdot)$ denotes the normal cone to $C(t, u(t))$ and $F(t, u(t))$ is a multifunction. Problem $(\mathcal{P})$ includes as a special case the following evolution quasi-variational inequality: Find $u : [0, T] \rightarrow H, u(0) = u_0 \in K(u_0)$, such that $u(t) \in K(u(t))$ for all $t \in [0, T]$, and

$$\langle l(t), w - u(t) \rangle \leq \langle \dot{u}(t), w - u(t) \rangle + a(u(t), w - u(t)) + j(w) - j(u(t)) \quad (1)$$

for all $w \in K(u(t))$.

Here $a(\cdot, \cdot)$ is a real bilinear, symmetric, bounded, and elliptic form on $H \times H, l \in H^{1,2}([0, T]; H)$, and $j(\cdot)$ denotes a non-negative, convex, positively homogeneous and Lipschitz continuous functional from H to $\mathbb{R}$. $K(u) \subset H$ is a set of constraints.

Keywords: variational inequality, sweeping process, evolution problem

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Strong A-summability of order alpha

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Abstract

In this paper we investigate the concept of A-statistical convergence of order alpha and Strongly A-summability of order alpha for a non-negative regular summability matrix A . For $0 < \alpha \leq \beta \leq 1$ we prove that A-statistical convergence of order alpha implies A-statistical convergence of order β . Also we give some conditions under which strong A-summability of order α implies strong A-summability of order β for $0 < \alpha \leq \beta \leq 1$. Finally for $0 < \alpha \leq \beta \leq 1$ we prove that for bounded sequences strong A-summability of order α implies strong A-summability of order β .

Keywords: Statistical convergence, A-statistical convergence, strong A-summability.

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Extensions of I. Schur's Inequality for the Leading Coefficient of Bounded Polynomials with One or Two Prescribed Zeros

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Abstract

In [3] (see also [1, p. 679]), I. Schur determined the sharp upper bounds for the leading coefficient a_n of a real polynomial P_n of degree $\leq n$ from $\mathbf{B}_n$ assuming that P_n additionally has a zero at one endpoint or at both endpoints of the interval $\mathbf{I} = [-1, 1]$, where

$$\mathbf{B}_n = \{P_n : P_n(x) = \sum_{k=0}^n a_k x^k (n \geq 2, a_k \in \mathbb{R}) \text{ and } \|P_n\|_\infty = \sup_{x \in \mathbf{I}} |P_n(x)| \leq 1\}.$$

We provide a twofold extension of Schur's inequality by determining the exact majorants for all coefficients a_k (including a_n) of P_n from the encompassing convex set $\mathbf{C}_n$ (in place of the unit ball $\mathbf{B}_n$), given by

$$\mathbf{C}_n = \{P_n : P_n(x) = \sum_{k=0}^n a_k x^k (n \geq 2, a_k \in \mathbb{R}) \text{ and } |P_n(x_{n,i}^*)| \leq 1 \text{ for } 0 \leq i \leq n\},$$

and assuming that P_n has a zero at one endpoint or at both endpoints of $\mathbf{I}$.

Here the points $x_{n,i}^*$ are the alternation points on $\mathbf{I}$ of the n -th Chebyshev polynomial T_n of the first kind [2]. Solutions of various classical extremal problems for polynomials have been extended in this way: by transferring the problem setting from $\mathbf{B}_n$ to the superset $\mathbf{C}_n$ [1, pp. 672], [2, pp. 107].

Schur's inequalities resemble P. L. Chebyshev's coefficient inequality [1, (16.3.2)], whereas our extensions resemble V. A. Markov's coefficient inequalities [1, (16.3.4)]. We furthermore extend Schur's inequalities to sharp G. Szegő - type inequalities [1, (16.3.8)]. In order that, in the symmetric case $P_n(-1) = P_n(1) = 0$, Schur's extremal polynomial $T_n(x \cos(\pi/2n)) \in \mathbf{B}_n$ then retains its extremal property we have to modify $\mathbf{C}_n$ to $\mathbf{C}_n^*$ by changing $x_{n,i}^*$ ($1 \leq i \leq n-1$) to $x_{n,i}^* / \cos(\pi/2n)$.

Keywords: Chebyshev, coefficient, estimate, extension, extremal, inequality, leading, V. A. Markov, polynomial, prescribed, problem, Schur, Szegő, zero.

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An Example of Optimal Nodes for Interpolation Revisited

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Abstract

A famous unsolved problem in the theory of polynomial interpolation is that of explicitly determining a set of nodes which is optimal in the sense that it leads to minimal Lebesgue constants. In [1] a solution to this problem was presented for the first non - trivial case of cubic interpolation, see also [2]. We add here that the quantities that characterize optimal cubic interpolation can be compactly expressed as real roots of certain cubic polynomials with integral coefficients. This facilitates the presentation and impartation of the subject - matter and may guide extensions to optimal higher - degree interpolation.

Keywords: Bernstein conjecture, cubic, interpolation, Lebesgue constant, Lebesgue function, minimal, nodes, optimal, polynomial, root.

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Boundary value problems for impulsive fractional differential equations with non-local conditions

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Abstract

In this study, we discuss some existence results for the solutions to impulsive fractional differential equations with non-local conditions by using several fixed point theorems.

Keywords: Caputo fractional derivative, impulsive differential equation, existence and uniqueness, fixed point theorem.

References

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Fundamental eigenvalues of biharmonic equations on circularly periodic domains

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Abstract

Biharmonic equations have many applications, especially in fluid and solid mechanics. Applications in solid mechanics include buckling and vibration of plates which has extensive practices in civil, mechanical, and aerospace engineering as well as vibration of piezoelectric and acoustic devices. In this work, applications to vibrating plates are considered. The governing equation of a vibrating plate is given by the biharmonic equation $D\nabla^4 w + \rho w_{tt} = 0$ where w is displacement, D is flexural rigidity, and ρ is density. The biharmonic eigenvalue problem has a general analytical solution in a circular domain [1]. However, the difficulty in finding solutions arises when the domain is no longer circular. For rectangular domains, Navier's and Levy's series solutions for certain boundary conditions are known [1]. Therefore, fundamental frequencies of vibrating plates have been determined only for a limited class of plate geometries. The lack of analytical solutions for other domains with various boundary conditions led many researchers to use numerical methods. The Rayleigh-Ritz method, the finite element method, and the Galerkin's method are among such numerical methods. However; in some cases, the numerical methods often encounter the problem of singularity, scaling, and sensitivity to the boundary conditions. Some singularities for annular plates are pointed out by C.Y. Wang and C.M. Wang [2]. This is a collection of work [3, 4, 5] to developed a special formulation of perturbation method to improve accuracy and reliability of fundamental frequencies of circularly periodic plates. The purpose of the present work is to provide approximate analytical formulation of the fundamental frequency for clamped and simply supported plates with circularly periodic boundaries, especially plates with a core where singularities arise. We develop a boundary perturbation method to extract the fundamental eigenvalue of the biharmonic boundary value problem.

Keywords: Biharmonic equation, fundamental frequency, vibration, plates.

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Differential MAC Models in Continuum Mechanics and Physics

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Abstract

The method of additional conditions or MAC was applied to create an integro-differential equation of the membrane problem [1]. This problem was presented at the Conference AMAT-2008. Another method can be used to create the differential MAC model of the same membrane problem. The obtained differential equation is much more easier to analyze and to obtain the exact solutions of the problem. Similar partial differential equation is considered in [2] but the exact solutions in our case are not given there.

The method to create the differential MAC models in mathematical physics is as follows. The classically stated problem is taken. Then the particular test problem is considered which solution could be compared with an experimental solution. For example we can take a circular elastic membrane with the fixed boundary condition at the contour and with the finite displacement in the center of membrane. The approximate experimental solution could be a cone. Substituting this solution into the classical membrane equation we will find the term which does not allow to satisfy the equation. We exclude this term from the equation and so the differential equation of the MAC model is created. We do not do anything except to correct mathematical model using an experiment.

It should be noted that mathematically similar test problems exist in the linear isotropic theory for cylinder and in the fluid mechanics for the Hagen-Poiseuille flow for a pipe. Then the differential MAC models for linear isotropic elasticity and for Navier-Stokes equations will be created.

The following differential MAC models are presented too: tension of an elastic rod, elastic string, beam, plate, heat conduction equation, Maxwell's equations, Schroedinger equations, Klein-Gordon equation.

Keywords: MAC model, mathematical physics, elasticity.

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Direct results on the q -mixed summation integral type operators

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Abstract

In this study, we introduce a q -mixed summation integral type operators and investigate their approximation properties. We obtain a Voronovskaja type theorem and give direct results on degree of approximation for continuous functions.

Keywords: q -integral, q -mixed operators, Voronovskaja type theorem, K -functional, weighted approximation.

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Numerical Solutions of Nonlinear Second-Order Two-Point Boundary Value Problems Using Half-Sweep SOR With Newton Method

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Abstract

In this paper, we examine the performance of Half-Sweep Successive Over-Relaxation (HSSOR) iterative method together with Newton scheme namely Newton-HSSOR in solving the nonlinear systems generated from second-order finite difference discretization of the nonlinear second-order two-point boundary value problems. As well known that to linearize nonlinear systems, the Newton scheme has been used to transform the nonlinear system into the form of linear system. Then the basic formulation and implementation of Newton-HSSOR iterative methods are also presented. Numerical results for three test examples have demonstrated the performance of Newton-HSSOR method compared to other existing SOR methods

Keywords: Newton Scheme, Half-Sweep SOR Iteration, Second-Order Scheme, Nonlinear Two-Point Boundary Value Problem.

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L_p – Saturation Theorem for an Iterative Combination of Bernstein-Durrmeyer Type Polynomials

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Abstract

Gupta and Maheshwari [2] introduced a new sequence of Durrmeyer type linear positive operators P_n to approximate p –th Lebesgue integrable functions on $[0, 1]$. It is observed that these operators are saturated with $O(n^{-1})$. In order to improve the rate of approximation we consider an iterative combination $T_{n,k}(f; t)$ of the operators $M_n(f; t)$. This technique was given by Micchelli [3] who first used it to improve the order of approximation by Bernstein polynomials $B_n(f; t)$.

In our paper [1] we obtained direct theorems in ordinary approximation in the L_p – norm by the operators $T_{n,k}$. Subsequently, we [4] proved a corresponding local inverse theorem over contracting intervals. The object of the present paper is to continue this work by proving the saturation theorem in a local set-up.

Keywords: linear positive operators, Bernstein-Durrmeyer Type Polynomials, iterative combination.

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Existence and uniqueness of the common tripled fixed point in generalized metric spaces

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Abstract

In this paper we prove a unique common tripled fixed point theorem under W-compatibility condition for two mappings in a generalized metric space.

Keywords: G-Metric Spaces, W-Compatible maps, Tripled fixed point.

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Generalized sampling with multi filterings

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Abstract

Sampling is the process of representing a continuous time signal by a discrete set of measurements, which are usually sample values of a signal at some instances. Using discrete sample values is ideal but in a general sampling scheme, nonideal samples can be given as inner products of a signal with a set of sample functions associated with the acquisition devices. Here we consider the problem of reconstructing any signal $f(t)$ of finite energy, that is, $f(t)$ in $L^2(R)$ from samples of responses of several linear time invariant systems.

The reconstructed signal $\tilde{f}(t)$ of $f(t)$ is sought in the shift invariant space $V(\Phi)$ with multi generators in $L^2(R)$. Hence in general, we can not expect the exact reconstruction of $f(t)$ but we seek an approximation $\tilde{f}(t)$, which is consistent with the input signal in the sense that it produces exactly the same measurements as the original input signal when it is reinjected into the system. It means that $f(t)$ and $\tilde{f}(t)$ are essentially the same to the end users, who can observe signals only through the given measurements.

We also discuss the performance analysis of the proposed generalized sampling scheme.

Keywords: Generalized sampling, consistency.

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Estimation of hazard function in continuous semi-Markov multi-state models

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Abstract

Semi-Markov multi-state stochastic processes are very important to describe regression and progression chronic diseases and cancers. In this research, a multi-state model with four states (state1: bone marrow transplantation, state2: chronic graft versus host disease (cGvHD), state3: platelet recovery, state4 (absorb state): death was defined. Weibull distribution density function was chosen as appropriate density function for the transition time between states. In the semi-Markov multi-state models The effect of sojourn time in the states on survival time was considered. A total of 507 acute leukemia patients (206 acute lymphocyte leukemia (ALL), 301 acute myeloid leukemia (AML)) at Shariati Hospital, Tehran, Iran were selected. The median of follow up time was 1.5 year. Coefficient of Weibull distribution in semi-Markov multi-state model showed that with increasing sojourn time in state1, hazard of cGVHD increased. Results show that the effects of some covariates were not constant during disease time, for example; after platelet recovery, Death hazard of acute lymphoblastic leukemia patients was 2.15 times of acute myeloid leukemia patients. There was a negative correlation between sojourn time in state 1 and state2 ($r = -.17, P < .001$). Multi-state semi-Markov models are useful models to accommodate multiple events and time dependent covariates.

Keywords: Semi-Markov; Multi-state ;stochastic process; hazard function; Leukemia.

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Connection between solutions nonlinear Schrodinger equation and spin system

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Abstract

We consider 2+1 dimensional nonlinear Schrodinger equation type

$$iq_t + M_1 q + vq = 0, \quad ir_t - M_1 r - vr = 0, \quad M_2 v = -2M_1(rq), \quad (1)$$

where q , r and v ($v = 2(U_1 - U_2)$) are some complex functions. The operators M_1 and M_2 are defined by

$$M_1 = 4(a_2 - 2ab - b)\partial_{xx}^2 + 4\alpha(b - a)\partial_{xy}^2 + \alpha^2\partial_{yy}^2, \quad (2a)$$

$$M_2 = 4a(a + 1)\partial_{xx}^2 - 2\alpha(2a + 1)\partial_{xy}^2 + \alpha^2\partial_{yy}^2, \quad (2b)$$

where a , b are arbitrary real constants, α is a complex constant.

Also consider the spin system

$$iS_t + \frac{1}{2}[S, M_1 S] + A_2 S_x + A_1 S_y = 0, \quad (3a)$$

$$M_2 u = \frac{\alpha^2}{2i} \text{tr}(S[S_x, S_y]), \quad (3b)$$

where S is a spin vector, M_1 , M_2 operators are given by (2).

$$A_1 = i\alpha(2b + 1)u_y - 2(2ab + a + b)u_x,$$

$$A_2 = i4\alpha^{-1}(2ab + a^2 + 2ab + b)u_x - 2(2ab + a + b)u_y.$$

Nonlinear Schrodinger equation type (1) and spin system (3) are equivalent, more exactly, gauge equivalent to each other in the paper [1,2]. Now using a $\bar{\partial}$ -problems method will find solutions of these systems (1) and (3).

Keywords: Nonlinear partial differential equation, spin system, nonlinear Schrodinger equation

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An iterative regularization method for a class of inverse problems for elliptic equations with Dirichlet conditions

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Abstract

In this talk, we investigate the inverse problem of finding the source in an abstract second-order elliptic equation on a finite interval. The additional information given is the value of the solution at an interior point of the interval. We prove existence, uniqueness, and the convergence results of KMF-iterative regularization method applied to the considered inverse problem.

Keywords: Ill-posed problems, inverse problems, KMF-iterative regularization.

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Constructions of determinantal representation of trigonometric polynomials

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Abstract

Let A be an $n \times n$ matrix. The numerical range of A is defined as the set $W(A) = \{\xi^* T \xi : \xi \in \mathbf{C}^n, \xi^* \xi = 1\}$. A ternary homogeneous polynomial associated with A defined by $F_A(t, x, y) = \det(tI_n + xH + yK)$ is hyperbolic with respect to $(1, 0, 0)$, where $H = (A + A^*)/2$, $K = (A - A^*)/(2i)$ are real and imaginary parts of A . It is well known that $W(A)$ is the convex hull of the real affine part of the dual curve of the curve $F(t, x, y) = 0$. The Fiedler-Lax conjecture is recently affirmed, namely, for any real ternary hyperbolic form $F(t, x, y)$, there exist real symmetric matrices S_1 and S_2 such that $F(t, x, y) = \det(tI_n + xS_1 + yS_2)$. In this talk, we give constructive proofs of the existence of real symmetric matrices for the ternary forms associated with trigonometric polynomials using Bezoutian and Sylvester elimination methods.

Keywords: Numerical range, determinantal representation, Bezoutian, Sylvester elimination.

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On Properties of the NODE System Connected with Cluster Traffic Model

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Abstract

Following-the-leader model of one-dimensional totally connected moving chain, [1], was formulated in the middle of 50-s

$$x_{n-l}(t) < \dots < x_{n-1}(t) < x_n(t) < x_{n+l}(t) < \dots < x_{n+k}(t), \quad (1)$$

where $x_n(t)$ is the coordinate of n -th particle,

$$x_{n+1}(t) - x_n(t) = f(\dot{x}_n(t)). \quad (2)$$

If f is a linear function, then a system has a exact solution and is completely investigated. In the case of non-linear dependence there are not enough correct statements and strictly researched problems, [2]. Physical way of knowledge disposed to transition of limit in particles quantity and reduction of systems of nonlinear ordinary differential equations (NODE) to partial derivative equations, that not satisfactorily describing all physical processes, and, partially, traffic, according to experimental data, [3].

Logical contradictions consist in that the density per kilometer is piecewise constant with the step of digitization of the order $10^{(-2)}$ and with the piecewise linear smoothing folds-insertions has a piecewise discontinuous derivative on the coordinates, which is, generally speaking, leads to incorrect reduction to the equation in partial derivative. Alternative moving to the density in the model (2) is the separation in (1) of stable clusters moving with the same speed. If f is a monotonically increasing function, then from (2) it follows that the cluster consists of equidistant from the neighbors of particles, where distances between particles are defined by leader velocity.

Problem statement. Cluster model of partial flow is a sequence of coordinates (1), where $x_i(t)$ is a boundary in cluster's separation, $y_i(t)$ is a density of particles flow on interval $[x_i, x_{i+1}]$, $f(y_i)$ is a cluster speed. We have the following NODE systems

$$\{\dot{x}_{i+1} = (f(y_{i+1})y_i + 1 - f(y_i)y_i)/(y_{i+1} - y_i); \quad y_i(t) \equiv y_i\};$$

or

$$\{\dot{x}_i = f(y_i); \quad \dot{y}_i = y_i(f(y_{i+1}) - f(y_i))/(x_{i+1} - x_i); \quad \dot{x}_{i+1} = f(y_{i+1})\}.$$

Multilane traffic model. There are considered possible formalizations of interaction of clusters located on adjacent lanes, lane changing and ramps.

Keywords: System of nonlinear differential equations, theory of traffic flow, following-the-leader model, cluster model of particles movement.

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Alternative variational iteration method to solve the time-fractional Fornberg-Whitham equation

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Abstract

In (Odibat, 2010), Odibat proposed an alternative approach of variational iteration method. In this paper, we applied alternative variational iteration method (AVIM) to solve time-fractional Fornberg-Whitham equation. We also compare the results with variational iteration method (VIM). The fractional derivatives are taken in the Caputo sense. The present methods performs extremely well in terms of efficiency and simplicity. Numerical results for different particular cases of the problem are presented.

Keywords: Alternative variational iteration method, Time-fractional Fornberg-Whitham equation, Caputo derivative, Auxiliary parameter, Variational iteration method.

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A Method of Solution for Integro-Differential Parabolic Equation with Purely Integral Conditions

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Abstract

The objective of this paper is to prove existence, uniqueness, and continuous dependence upon the data of solution to a integrodifferential parabolic equation with only integral conditions. The proofs are based on a priori estimates and Laplace transform method. Finally, we obtain the solution by using a numerical technique for inverting the Laplace transforms.

Keywords: Laplace Transform, Integrodifferential parabolic equation, Integral conditions.

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A Related Fixed Point Theorem in n -Intuitionistic Fuzzy Metric Spaces

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Abstract

We prove a related fixed point theorem for n mappings in n Intuitionistic fuzzy metric spaces using an implicit relation which generalizes results of Aliouche and Fisher [1], Merghadi and Aliouche [3] and Rao et al. [4].

Recently, Merghadi and Aliouche [3] Aliouche and Fisher [1], Aliouche et.al [2] and Rao et.al [4] proved some related fixed point theorems in compact metric spaces and sequentially compact fuzzy metric spaces. Motivated by a work due to Popa [5], we have observed that proving fixed point theorems using an implicitly relation is a good idea since it covers several contractive conditions rather than one contractive condition.

Keywords: Fuzzy metric space; implicit relation; Intuitionistic fuzzy metric space; related fixed point.

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A Modified Partial Quadratic Interpolation Method for Unconstrained Optimization

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Abstract

A numerical method for solving large-scale unconstrained optimization problems is presented. It is a modification of the partial quadratic interpolation method [1] for unconstrained optimization and based upon approximating the gradient and the Hessian of the objective function. This means that it requires only the expression of the objective function to converges to a stationary point of the problem from any initial point, with speed convergence. The method can solve complex problems in which direct calculations of the gradient and Hessian matrix are difficult or even impossible to calculate. The search directions are always descent directions. Results and comparisons are given at the end of the paper and show that this method is interesting.

Keywords: Unconstrained optimization, descent direction, partial quadratic interpolation method.

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Skewed Bimodal Laplace Distribution

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Abstract

New Skew Bimodal Laplace Distribution is proposed. Characterization studies of the distribution will be conducted. A simulation study will be conducted and then compared to the currently used competitive distributions to assess the performance of the model.

Keywords: Skew Bimodal Distribution, bimodality parameter, Threshold Parameter, Bimodality.

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Basic Results of Nonlinear Eigenvalue Problems of Fractional Order

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Abstract

In this paper, basic theory of boundary value problems of fractional order $1 < \delta < 2$ involving the Caputo derivative is discussed. By applying the maximum principle, necessary conditions for the existence of eigenfunctions are obtained, as well as, analytical lower and upper bounds estimates of the eigenvalues. A sufficient condition for the non existence of ordered solution is obtained by transforming the problem into equivalent integro-differential equation. By means of the method of lower and upper solution we obtain a general existence and uniqueness result of the problem. We generate two well defined monotone sequences of lower and upper solutions which converge uniformly to the actual solution of the problem. While some fundamental results are obtained, we leave others as open problems and report them as a conjecture.

Keywords: Fractional differential equations, Boundary value problems, Maximum principle, Lower and upper solutions, Caputo fractional derivative.

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A computational method for solving a class of non-linear fourth order singularly perturbed boundary value problem

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Abstract

In this paper, a computational method is presented for solving a class of fourth-order singularly perturbed boundary-value problems with a boundary layer at one end. The implemented technique consists of solving two problems which are a reduced problem and a boundary layer correction problem. The pade'approximation technique is used to satisfy the conditions at infinity. Theoretical and numerical results are presented.

Keywords: Fourth-order, singularly perturbed boundary-value problems, boundary layer, boundary layer correction, pade'approximation.

Exchangable Parameters Binomial Approximation

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Abstract

Present at the point which modern science, used mathematical analysis methods in statistical analysis has commonly become inevitable. In particular, as depending on the development of mathematical methods in mathematical statistics, concepts and features of probabilistic in non-linear regression analysis was made major contributions in the development of new methods and theoretical findings. Mathematical analysis to study these important materials by taking polynomials, Bernstein polynomials and important finding presented in the last few years in which these results are being referred to the features highlighted by the statistical analysis used. In this paper, the statistical results of Bernstein approaches have resulted in more efficient in non-linear regression models. Especially if the number of observations remains xed, used iterative process in Bernstein polynomial lowered faster than margin of errors.

In this study it is intended an approximation thoroughly *Exchangable parameters* Binomial distribution In this approach the exchangable parameters binomial distribution was used instead of Bernstein polynomial kernel and was investigated statistical efficiency in non linear model

Keywords: Non Linear Model, Bernstein Approach, Exchangable Parameter Binomial Distribution.

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On the boundedness and the stability properties of solution of certain third order differential equations

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Abstract

In this paper, we investigate equation (1) in two cases: (i) $P \equiv 0$, (ii) $P(\neq 0)$ satisfies $|P(t, x, y, z)| \leq (A + |y| + |z|)q(t)$, where $q(t)$ is a nonnegative function of t . For case (i) the stability of the solution $x = 0$ and the uniform boundedness of all solutions are investigated and for case (ii) the boundedness result is obtained for solutions of equation (1). These results improve and include several well-known results. The differential equation considered is of the form

$$\ddot{x} + \psi(t, x, \dot{x})\ddot{x} + f(t, x, \dot{x}) = P(t, x, \dot{x}, \ddot{x}). \quad (1)$$

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Some Extensions of Sufficient Conditions for Univalence of an Integral Operator

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Abstract

In this paper we introduce and study a general integral operator defined on the class of normalized analytic function in the punctured unit disk. This operator is motivated by many researchers. With this operator univalence conditions for the normalized analytic function in the open unit disk are obtained. In deed, we present a few conditions of univalency for our integral operator. The operator is essential to obtain univalence of a certain general integral operator. Having the integral operator, there are interesting properties of normalized function in the unit disk for univalent conditions for an integral operator. In addition, various other known results are also pointed out. We also find some interesting corollaries on the class of normalized analytic of functions functions in the open unit disk. Our results certainly generalized several results obtained earlier. Therefore, many interesting results could be obtained and we also derive some interesting properties of these classes. The operator defined can be extended and can solve many new results and properties. The work presented here is the generalization of some work done by earlier researchers. For example, see [1, 2].

Keywords: Analytic functions, univalent functions, integral operator, univalent conditions, schwarz Lemma.

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Performance Evaluation of Object Clustering using Traditional and Fuzzy Logic Algorithms

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Abstract

This work analyzes three algorithms for object clustering. The algorithms are: the k-means, fuzzy c-means (FCM), and kernel fuzzy c-means (KFCM). The k-means algorithm is a crisp method that partitions a dataset into hard clusters. Both FCM and KFCM are based on fuzzy logic and they return a degree of membership of each object to all clusters. For evaluating their performance; the algorithms are implemented and run on two different datasets. As a conclusion; it is shown that the KFCM algorithm can achieve better results than the other two algorithms. The FCM is slightly better than the k-means algorithm. Moreover, the clustering time of each algorithm is different from the others. The clustering time of KFCM was larger than both the k-means and FCM. Also, clustering using the k-means has the smallest clustering time. Several parameters have significant effects on the overall performance. This involves the dimensionality value (i.e. the number of properties of each object), number of objects, number of iterations, number of clusters, fuzziness parameter, and the number of updating operations for both cluster centroids and membership values.

Keywords: Crisp Clustering, Fuzzy Clustering, Datasets, Algorithms, and performance Evaluation.

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A Better Error Estimation On Mixed Summation-Integral Operators

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Abstract

In the present paper, we study a King [9] type modified sequence of mixed summation-integral type operators, by this modification we give approximation properties and better approximation for these operators. Then we study the rate of convergence, Voronovskaya results and Korovkin theorem.

Keywords: Szász, Baskakov, Korovkin theorem.

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Boussinesq equation with a non classical condition

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Abstract

This paper deals with the solvability and uniqueness of a higher dimension mixed non local problem for a Boussinesq equation. The uniqueness and existence of a generalized solution is proved with the help of an a priori estimate and the galerkin approximation method, respectively.

Keywords: Nonlocal condition, a Priori estimate, Galerkin's method, Boussinesq equation.

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About New Class of Volterra Type Integral Equations with Boundary Singularity in Kernels

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Abstract

Let $\Gamma = \{x : a < x < b\}$ the set of point on real axis and consider an integral equation

$$\varphi(x) + \int_a^x [K_1(x, t) + K_2(x, t) \ln(\frac{x-a}{t-a})] \frac{\varphi(t)}{t-a} dt = f(x), \quad (1)$$

where $K_1(x, t), K_2(x, t)$ are given functions on the rectangle $\overline{R}$, with R defined as the set $\{a < x < b, a < t < b\}$, $f(x)$ are given function in $\overline{\Gamma}$ and $\varphi(x)$ to be found.

The theory of the integral equation (1) at $K_2(x, t) = 0$ has been constructed in [1]. In this work based on the roots of the algebraic equation $\lambda^2 + K_1(a, a)\lambda + K_2(a, a)$, signs $K_1(a, a), K_2(a, a)$ the general solution of the model integral equation (1) in explicit form is obtained. Moreover, using the method similar to regularization method [1] in theory one-dimensional singular integral equation, the problem of finding general solution of integral equation (1), is reduced to the problem of finding general solution of integral equation with weak singularity.

In particular, for equation (1) the following confirmation is obtained.

Theorem 1. Let in integral equation (1) $K_1(x, t) = p = \text{const.} < 0, K_2(x, t) = q = \text{const.} < 0, p^2 > 4q, f(x) \in C[a, b], f(a) = 0$ with the following asymptotic behavior

$$f(x) = o[(x-a)^{\delta_1}], \quad \delta_1 > \lambda_1, \quad \lambda_1 = \frac{|p| + \sqrt{p^2 - 4q}}{2} \quad \text{as } x \rightarrow a.$$

Then integral equation (1) in class function $\varphi(x) \in [a, b]$, vanishing in point $x = a$, always solvability and its solution is given by the following formula

$$\begin{aligned} \varphi(x) = & (x-a)^{\lambda_1} c_1 + (x-a)^{\lambda_2} c_2 + f(x) - \frac{1}{\sqrt{p^2 - 4q}} \int_a^x [\lambda_2^2 (\frac{x-a}{t-a})^{\lambda_2} - \\ & - \lambda_1^2 (\frac{x-a}{t-a})^{\lambda_1}] \frac{f(t)}{t-a} dt \equiv K_1^-[c_1, c_1, f(x)], \end{aligned}$$

where c_1, c_2 -arbitrary constants, $\lambda_2 = \frac{|p| - \sqrt{p^2 - 4q}}{2}$.

Keywords: Integral equation, Singular kernels.

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Nine point multistep methods for linear transport equation

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Abstract

In this paper we construct a family of multistep methods on a nine point square by collocating for the linear advection equation. Square polynomials are used for this purpose. Numerical examples show the performance of the different methods according to the choice of the parameters.

Keywords: Linear transport equation, finite difference methods, multistep methods, advection-diffusion equation.

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Testing problems for sparse contingency tables

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Abstract

Recently amounts of information are very extensive, therefore issues related to a large dimension and sparsity of (categorical) data arise rather frequently and are widely discussed in the literature [1, 9]. According to a rule of thumb, a contingency table is sparse if expected (under the null hypothesis) frequencies in a significant part of the contingency table are less than 5 [7]. For sparse contingency tables, the χ^2 approximation to the distribution of goodness-of-fit statistics may be inaccurate.

Several techniques have been proposed to tackle this problem: exact tests [2, 5] and alternative approximations [6], the parametric and nonparametric bootstrap [4], Bayesian approach [3], and other methods. However, for (very) sparse categorical data, common goodness-of-fit tests may be inconsistent [8] and hence there is no sense to approximate their distributions.

In the presentation alternative goodness-of-fit tests based on categorical data smoothing are proposed. The smoothing procedure uses the clusterization and nonparametric estimation of the contingency table profile. The performance of the tests is illustrated by computer simulations.

Keywords: sparse contingency table, goodness-of-fit, chi-square, profile statistic.

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Fractional integration of the product of two H-functions and a general class of polynomials

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Abstract

The aim of the present paper to study and develop the generalized fractional integral operators recently introduce by Saigo [5]. First, author establish two results that give the image of the products of two H-functions and a general class of polynomials in Saigo operators. These results, besides being of very general character have been put in a compact form avoiding the occurrence of infinite series and thus making them useful in applications. Our findings provide interesting unifications and extensions of a number of (new and known) images. For the sake of illustration, we give here exact references to the results (in essence) of eight research papers [1, 2, 3, 4, 7, 8, 10 and 11] that follow as particular cases of our findings.

Keywords: Fractional integral operators by Saigo, Riemann-Liouville and Erdelyi-Kober, H-function of severables variables, general class of polynomials Mittag-Leffer functions.

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Comparing the Box - Jenkins models before and after the wavelet filtering in terms of reducing the orders with application

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Abstract

In this paper, the estimated linear models of Box-Jenkins such as AR(p), MA(q) and ARMA(p,q) has been compared from time series observations, before and after wavelet shrinkage filtering (used to solve the problem of contamination (or noise) if it found in the observations) and then reducing the order of the estimated model from filtered observations (with preserving the accuracy and suitability of the estimated models) and re-compared with the estimated linear models of original observations, depending on some statistical criteria, including the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), and the Akaike's Information Criterion (AIC), through taking a practical application of time series consistent with the models mentioned above and using statistical programs such as Statgraphics, NCSS and MATLAB.

The results of the paper showed the efficiency of wavelet shrinkage filters in solving the noise problem and obtaining the efficient estimated models, and specifically the wavelet shrinkage filter (dmey) with Soft threshold which estimated its level using the Fixed Form method of filtered observations, and the possibility of obtaining linear models of the filtered observations with lower orders and higher efficiency compared with the corresponding estimated models of original observations.

Reduced bias of the mean for a heavy-tailed distribution

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Abstract

In this paper, we deal with bias reduction techniques for estimate the mean for a heavy tailed distribution with index $1/2 < \gamma < 1$, the semi-parametric estimation of mean depends not only on the estimation of the extreme value index γ , the primary parameter of extreme events, but also on the adequate estimation of a scale first order parameter. Recently, apart from new classes of reduced-bias estimators for $\gamma > 0$ and of the quantile extreme, new classes of the scale first order parameter have been introduced in the literature. Their use in mean estimation enables us to introduce new classes of asymptotically unbiased mean estimators, with the same asymptotic variance as the (biased) “classical” estimator. The asymptotic distributional behaviour of the proposed estimators of μ is derived, under a second-order framework, and their finite sample properties are also obtained through Monte Carlo simulation techniques.

Keywords: Heavy tails, mean, trimmed mean, Hall class, Statistics of Extremes.

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Statistical Approximation of Truncated Operators

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Abstract

Linear positive operators and their Korovkin type statistical approximation properties have been investigated by many mathematicians until today. It is well-known that lots of operators were defined with infinite series. On the other hand, we can investigate the statistical approximation properties considering only the partial sums of the operators.

We investigate the statistical approximation properties of the operators which was built and examined the ordinary approximation properties by Agratini in [1]. Then, we state the statistical convergence of the truncated operators which is related to that operators.

Keywords: Sequence of positive linear operators, Korovkin theorem for statistical approximation, Rate of convergence

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On The Number of Representations of An Integer of The Form $x^2 + dy^2$ in A Number Field

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Abstract

Let ω be an integer in a number field K and d a positive rational integer. We say that ω has representation of the form $x^2 + dy^2$ if there are integers α and β in K such that $\omega = \alpha^2 + d\beta^2$.

In this paper, we show that there are infinitely many representations of every integer of the form $x^2 + dy^2$ in any number field K except when $K = \mathbb{Q}(\sqrt{-d})$ or K is totally real.

Keywords: Integer, totally real, number field.

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On the Hyers-Ulam stability of non-constant valued linear differential equation $xy' = -\lambda y$

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Abstract

We consider a differentiable map y from an open interval to a real Banach space of all bounded continuous real-valued functions on a topological space. We will investigate the Hyers-Ulam stability of the following linear differential equations of first order with non-constant values:

$$xy' = -\lambda y,$$

where λ is a positive real number and

$$y \in C(I) = C(a, b), -\infty < a < b < +\infty, x \in (0, \infty).$$

Keywords: Hyers-Ulam Stability, Differential equation, Approximation.

The Approximate Solution of multi-higher Order Linear Volterra Integro-Fractional Differential Equations with Variable Coefficients in Terms of Orthogonal Polynomials

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Abstract

The main purpose of this paper is to present an approximation method for multi-higher order Linear Volterra Integro-Fractional Differential Equations (m-h LVIFDEs) with variable coefficients in most general form under the conditions. The method is based on the orthogonal polynomials (Chebyshev and Legendre) via least square technique. This method transforms the equation and the given conditions into matrix equations which correspond to a system of linear algebraic equations with unknown coefficients and apply Gaussian elimination method to determine the approximate orthogonal coefficients. The proposed method contains two new algorithms for solving our problem, for each algorithm a computer program was written. Finally, numerical examples are presented to illustrate the effectiveness and accuracy of the method and the results are discussed.

Keywords: Integro-fractional differential equation, Caputo fractional derivative, least-square technique, Orthogonal (Chebyshev and Legendre) polynomial.

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On Applications of Fractional Calculus Involving Summations of Series

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Abstract

In the present paper author derive a number of summations of series concerning generalized hypergeometric function which are applications of the one of Samko result. Samko provided extensions to the familiar Leibniz rule for the n th derivative of product of two functions.

Keywords: Fractional calculus, generalized Leibniz rule, generalized hypergeometric series.

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Comparing Some Robust Methods with OLS Method in Multiple Regression with Application

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Abstract

The classical method, Ordinary Least Squares (OLS) is used to estimate the parameters of the linear regression, when assumptions are available, and its estimators have good properties, like unbiasedness, minimum variance, consistency, and so on. The alternative statistical techniques have been developed to estimate the parameters, when the data is contaminated with outliers. These are the robust (or resistant) methods. In this thesis, three of robust methods are studied, which are: Maximum likelihood type estimate M-estimator, Modified Maximum likelihood type estimate MM-estimator and Least Trimmed Squares LTS-estimator, and their results are compared with OLS method. These methods applied to real data taken from the Charstin company for manufacturing furniture and wooden doors, the obtained results compared by using the criteria: Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE) and Mean Sum of Absolute Error (MSAE). The important conclusions that this study came up with are: the number of outlier values detected by using the four methods in the data for furniture's line are very close. This refers to the fact that the distribution of standard errors is close to the normal, but the outlier values found in the data for doors line, by using OLS are less than which detected by robust methods. This means that the distribution of standard errors is departure distant from the normal. The other important conclusion is that estimated values of parameters by using OLS are very far from its estimated values by using the robust methods with respect to doors line, the LTS-estimator gave better results by using MSE criterion, and M-estimator gave better results by using MAPE criterion. Further more, it has noticed that by using the criterion MSAE, the MM-estimator is better. The programs S-plus (version 8.0, professional 2007), Minitab (version 13.2) and SPSS (version 17) are used to analyze the data.

The norm estimates of the q -Bernstein operators in the case $q > 1$

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Abstract

During the last decade, q -Bernstein polynomials have been brought to the spotlight and studied by a number of authors from different angles. While for $q = 1$ these polynomials coincide with the classical Bernstein polynomials, for $q \neq 1$ we obtain new polynomials with rather unexpected properties. A comprehensive review of the results on q -Bernstein polynomials, along with an extensive bibliography, is provided in [1]. It has been known that the q -Bernstein polynomials tend to retain some of the properties of the classical Bernstein polynomials: for example, they possess the end-point interpolation property, admit a representation via divided differences, demonstrate the saturation phenomenon, and reproduce the linear functions (see, e.g. [2]). Although establishing the similarity between the Bernstein and the q -Bernstein polynomials is essential for research, the study of the latter case is not restricted to merely drawing analogies between the classical polynomials and the q -versions.

The first example illustrating the essential differences in between the properties of the q -Bernstein polynomials and those of the classical ones is on the convergence properties. What is more, the cases $0 < q < 1$ and $q > 1$ in terms of convergence are not similar to each other. This absence of similarity is brought about by the fact that, for $0 < q < 1$, $B_{n,q}$ are *positive* linear operators on $C[0, 1]$, whereas for $q > 1$, no positivity occurs. In addition, the case $q > 1$ is aggravated by the rather irregular behavior of basic polynomials which, in this case, combine the fast increase in magnitude with the sign oscillations.

In this talk, the norm estimates in $C[0, 1]$ for the q -Bernstein basic polynomials and the q -Bernstein operator $B_{n,q}$ are presented. While, for $0 < q \leq 1$, $\|B_{n,q}\| = 1$ for all $n \in \mathbb{N}$, in the case $q > 1$, the norm $\|B_{n,q}\|$ increases rather rapidly as $q \rightarrow +\infty$. It is proved (see [3]) that $\|B_{n,q}\| \sim C_n q^{n(n-1)/2}$, $q \rightarrow +\infty$ with $C_n = \frac{2}{n} (1 - \frac{1}{n})^{n-1}$. Moreover, it is shown that

$$\|B_{n,q}\| \sim \frac{2q^{n(n-1)/2}}{ne} \text{ as } n \rightarrow \infty, q \rightarrow +\infty.$$

Keywords: q -Bernstein polynomials, q -Bernstein operator, operator norm, Newton's method.

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An Approximating Non-stationary Subdivision Scheme for Designing Curves

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Abstract

In this paper we present a non-stationary approximating subdivision scheme for designing curves. This scheme is a non-stationary counterpart of the known four-point stationary scheme. We show that the scheme is C^2 and reproduces some trigonometric functions. We highlight some advantages of the scheme over some existing stationary and non-stationary schemes. We also demonstrate its performance by some examples.

Keywords: Non-stationary subdivision, smoothness, approximation, curves.

Parallel Solution Schemes for Quasi-Tridiagonal Linear Systems Arising After Discrete Approximations of ODEs/PDEs with Nonlocal Conditions

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Abstract

We consider the system of linear algebraic equations $AY = F$, where A is a quasi-tridiagonal matrix (additional non-zero elements appear in the first and last rows of the matrix A), Y and F are unknown and given vectors, respectively. Such linear systems arise after discrete (e.g. finite-volume or finite-difference) approximations of ODEs/PDEs with nonlocal conditions [1, 2, 3]. Nonlocal conditions appear when the value of unknown function or its derivative on the boundary is related with the values inside the domain. Since problems with nonlocal conditions arise in various fields of chemistry, biology, physics, biotechnology, etc., the development of efficient tools for their numerical solution is essential. The main aim is to construct and investigate efficient parallel direct schemes for solution of the quasi-tridiagonal linear systems.

We construct and analyse two parallel solution schemes. Both schemes are based on the idea of splitting the considered quasi-tridiagonal system into three tridiagonal systems [4]. In order to solve these tridiagonal systems simultaneously, we employ the special parallel version of the well-known two-way Gaussian algorithm (twisted factorization algorithm) or Wang's method. The results of theoretical analysis of suggested parallel solution schemes are presented.

Keywords: Linear system, quasi-tridiagonal matrix, parallel computing, nonlocal conditions.

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Lower and Upper Estimate for Christoffel-function associated with a doubling measure on a quasismooth curve or arc¹

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Abstract

Recently G. Mastroianni and V. Totik have shown that the adjacent zeros of orthogonal polynomials associated with a doubling measure with support $[-1; 1]$ are uniformly spaced [1]. The application of weighted lower and upper estimate for Christoffel function [2] has got an important role in the construction of the upper estimate. In [3, 4] we investigated the zero-spacing on an interval where the measure has doubling property and an estimate for Christoffel function was again used at the upper estimate.

Most recently Andrievskii has generalized some weighted inequalities in [2] for quasismooth curve or arc in complex plane [5]. Using his approach we show a general estimate for Christoffel function associated with a doubling measure over quasismooth curve or arc. This estimate involves the interval case mentioned above.

Keywords: Christoffel-function, doubling measure, quasismooth curve or arc, fast decreasing polynomials.

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To Approximate Solution of Ordinary Differential Equations

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Abstract

We consider boundary value and initial value problems for ordinary differential equations.

1. Let us divide BVP into two classes. We include in the first class the problems satisfying the *Banach-Picard-Schauder* conditions and in the second class - BVP when they satisfy the Maximum Principle. In this direction the following statement is typical [1]:

Stmt.1 The order of arithmetic operations for calculation of approximate solution and its derivative of BVP for nonlinear second order DE or for system with two equations of normal form with Sturm-Liouville boundary conditions is $O(n \ln n)$ Horner unit. The convergence of the approximate solution and its derivative has $(p - 1)$ order with respect to mesh width $h = 1/n$ if $y(x)$ has $(p + 1)$ order continuously differentiable derivatives. If the order is less than p , the remainder member of corresponding scheme has best constant in Sard's sense.

We remark that in this case the basic apparatus are special spline-functions and special method of finite sums. These results refined and generalized corresponding results by Shrder, Collatz, Quarteroni-Buthcher-Stetter, having first order of convergence and AOS is $O(n^2 \ln n)$. First order of convergence with respect to n , where n is the number of subintervals, has the Multiple Shooting method (Keller, Osborne, Bulirsch) but the order of AOS is no less than $O(n^2)$.

For the second class of BVPs such typical results are true:

Stmt.2 Let us consider the BVP for linear second order DE (when the principal part has self-adjoint form too or contains small parameter) with Sturm-Liouville boundary conditions. Then new multi-point stable schemes are constructed by special spline-functions needing $O(1/h)$ of AOS, in such a way that the convergence of the approximate solution has $(p - 1)$ order with respect to h . When $p = 3$ this scheme is identical to classical ones. For $3 < p < 6$ these schemes are different from the Streng-Fix and Mikhlin unstable FEM.

2. With respect to numerical solution of Cauchy problem we used Gauss or Clenshaw-Curtis type quadratures and Hermite interpolation processes[1]. In this case:

Stmt.3 The multipoint schemes converge as $O(h^{2n})$ for any finite integer n and are absolutely stable when the matrices of nodes are normal types in Fejers sense.

Keywords: Ordinary differential equations (ODEs), initial and boundary value problems (BVP), approximate solution, mesh width, Horner unit, convergence, stable process, the order of arithmetic operations for finding approximate solution (AOS).

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Sigma Mass Dependence Of Static Nucleon Properties From Linear Sigma Model

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Abstract

The sensitivity of static nucleon properties (magnetic moment, axial-vector coupling constant g_A , pion-nucleon coupling constant $g_{\pi NN}$ and sigma commutator term $\sigma_{\pi N}$) to the quark and sigma masses have been investigated in the mean field approximation. I have solved the field equations in the mean field approximation with different sets of model parameters. Good results have been obtained in comparison with the other models and experimental data.

Keywords: Static Nucleon Properties, Linear Sigma Model.

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Approximation Techniques in Impulsive Control Problems for the Tubes of Solutions of Uncertain Differential Systems

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Abstract

The paper deals with the control problems for the system described by differential equations containing impulsive terms (or measures). The problem is studied under uncertainty conditions with set-membership description of uncertain variables [1], which are taken to be unknown but bounded with given bounds (e.g., the model may contain unpredictable errors without their statistical description). The main problem is to find external and internal estimates for set-valued states of nonlinear dynamical impulsive control systems and related nonlinear differential inclusions with uncertain initial state. Basing on the techniques of approximation of the generalized trajectory tubes by the solutions of usual differential systems without measure terms and using the techniques of ellipsoidal calculus [2, 3, 4] we present here a new state estimation algorithm for the studied impulsive control problem. The examples of construction of such ellipsoidal estimates of reachable sets and trajectory tubes of impulsive control systems are given. The applications of the problems studied here are in guaranteed state estimation for nonlinear systems with unknown but bounded errors and in nonlinear control theory.

Keywords: Impulsive control, uncertainty, ellipsoidal approximations.

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Optimal inequalities for linear functions of monotone sequences

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Abstract

Motivated by statistical applications, we present sharp lower and upper bounds for arbitrarily fixed linear combinations of properly centered arbitrary non-decreasing sequences of fixed length, expressed in various scale units. Positive upper bounds (and negative lower ones) are derived by means of projecting the linear function onto the convex cone of non-decreasing sequences in the standard Euclidean space. The remaining ones are established by solving a dual problem of maximizing the norm over a properly chosen convex set.

Keywords: Monotone sequence, linear functional, optimal bound.

Some Properties of q-Bernstein Schurer Operators

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Abstract

In this paper, the approximation properties of q-Bernstein Schurer operators $B_n^p(f; q; x)$ for $f \in C([0, p+1])$ are studied. The order of convergence of the operators in terms of Lipschitz class functionals and the first and second modulus of continuity are discussed.

Keywords: q-Integers, Bernstein operators, Modulus of continuity.

Prescribed asymptotic behavior of solutions of second-order nonlinear differential equations¹

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Abstract

We show that the nonlinear second order differential equation

$$(p(t)x')' + q(t)x = f(t, x, x'), \quad t \geq t_0 \geq 0$$

where $p \in C([t_0, \infty), (0, \infty))$, $q \in C([t_0, \infty), \mathbb{R})$ and $f \in C([t_0, \infty) \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$ has monotone positive solutions with prescribed asymptotic behavior at ∞ . Examples are given to illustrate the obtained results.

Keywords: Asymptotic behavior, Fixed point theory.

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Trigonometric Approximation of Signals (Functions) Belonging to Weighted $(L_p, \xi(t))$ – Class by Hausdor Means

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Abstract

Rhoades (2001) has obtained the degree of approximation of functions belonging to the weighted $(L_p, \xi(t))$ class by Hausdorff means of their Fourier series, where $\xi(t)$ is an increasing function. The first result of Rhoades generalizes the result of Lal (1999). In a very recent paper, Rhoades et. al. (2011) has obtained the degree of approximation of functions belonging to the $Lip\alpha$ class by Hausdorff means of their Fourier series and generalized the result of Lal and Yadav (2001). In this paper, authors have made some important remarks, namely, increasing nature of $\xi(t)$ alone is not sufficient to prove the results of Qureshi (1982), Lal(1999), Rhoades (2001) and Lal & Singh (2002) and the condition $1/\sin\beta = O(1/t^\beta)$, $1/n \leq t \leq \pi$ used by all these authors is not valid since $\sin t \rightarrow 0$ as $t \rightarrow \pi$. They have also suggested a modification in the definition of weighted $(L_p, \xi(t))$ class and leave an open question for determining a correct set of conditions to prove the results of Rhoades (2001). We note that the same types of errors are appearing in the papers of Lal (2004, 1999), Nigam (2010, 2011) and Nigam and Sharma (2011). Being motivated by the remarks of Rhoades et. al. (2011), in this paper, we determine the degree of approximation of functions belonging to the weighted $(L_p, \xi(t))$ class by Hausdorff means of their Fourier series and rectify the above errors by using proper set of conditions. Our paper is the improved version of the paper of Rhoades (2001), which in turn generalizes the result of Lal (1999). We also deduce some important corollaries from our results.

Keywords: Signal, Trigonometric Fourier approximation, Class $W(L_p, \xi(t))$, Hausdor means.

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Application of the hybrid method for the numerical solution of Volterra integral equations¹

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Abstract

It is known that there exists a class of methods for solving integral equations with variable boundary. Among them are the most popular methods of quadratures. This method is clarified and modified by many scholars. Here, to numerical solution of Volterra integral equations of the hybrid method is applied and constructed a concrete method with the degree $p = 6$ and $p = 8$, by using information about the solution of equations only one previous point.

Consider the numerical solution of the following nonlinear Volterra equation:

$$y(x) = f(x) + \int_{x_0}^x K(x, s, y(s)) ds, \quad x \in [x_0, X]. \quad (1)$$

We assume that (1) has a unique solution defined on the interval $[x_0, X]$. The first hybrid method for the solution of equation (1) constructed Makroglou [1] that in [2] is generalizable in the following form:

$$\sum_{i=0}^k \alpha_i y_{n+i} = \sum_{i=0}^k \alpha_i f_{n+i} + h \sum_{j=0}^k \sum_{i=0}^{j-\nu_i} \beta_i^{(j)} K(x_{n+j}, x_{n+l_i}, y_{n+l_i}) \quad (2)$$

$$(l_i = i + \nu_i, |\nu_i| < 1)$$

Method suggested here has the following form:

$$\sum_{i=0}^k \alpha_i y_{n+i} = \sum_{i=0}^k \alpha_i f_{n+i} + h \sum_{j=0}^k \sum_{i=0}^j \beta_i^{(j)} K(x_{n+j}, x_{n+i}, y_{n+i}) \quad (3)$$

$$+ h \sum_{j=0}^k \sum_{i=0}^{j-\nu_i} \gamma_i^{(j)} K(x_{n+j}, x_{n+l_i}, y_{n+l_i}).$$

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A Cauchy Problem for Some Local Fractional Abstract Differential Equation with Fractal Conditions ¹

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Abstract

Fractional calculus is an important method for mathematics and engineering [1-24]. In this paper, we review the existence and uniqueness of solutions to the Cauchy problem for the local fractional differential equation with fractal conditions

$$D^\alpha x(t) = f(t, x(t)), \quad t \in [0, T], \quad x(t_0) = x_0,$$

where $0 < \alpha < 1$ in a generalized Banach space. We use some new tools from Local Fractional Functional Analysis [25, 26] to obtain the results.

Keywords: Fractional analysis, local fractional differential equation, generalized Banach space, local fractional functional analysis.

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A new viewpoint to Fourier analysis in fractal space¹

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Abstract

Fractional analysis is an important method for mathematics and engineering [1-21], and fractional differentiation inequalities are great mathematical topic for research [22-24]. In the present paper we point out a new viewpoint to Fourier analysis in fractal space based on the local fractional calculus [25-58], and propose the local fractional Fourier analysis. Based on the generalized Hilbert space [48, 49], we obtain the generalization of local fractional Fourier series via the local fractional calculus. An example is given to elucidate the signal process and reliable result.

Keywords: Fourier analysis, fractal space, local fractional calculus, generalized Hilbert space, fractional-order complex mathematics.

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Approximate Solution of some BVP of 2Dim Refined Theories

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Abstract

Let us consider von Kármán-Mindlin-Reissner type Refined Theories (RT) in wide sense when thin-walled elastic structure $\Omega_h = D(x, y) \times (h^-, h^+)$ is isotropic, the boundary conditions on the $S^\pm$ surfaces have the form $\sigma_3^\pm = g^\pm$. We remind the explicit form of RT given by (2.47) of [1]. Now, if we will find σ_3 vector so as [1] :

$$\sigma_3 = \frac{z - h^-}{2h} g^+ - \frac{z - h^+}{2h} g^- + \sum_{s=1}^{\infty} \sigma_3^s(x, y) \left[P_{s+1}\left(\frac{z - h^*}{h}\right) - P_{s-1}\left(\frac{z - h^*}{h}\right) \right],$$

where h is the half thickness and h^* is the mid-surface. The form of RT and *Filon-Kirchhoff* type systems of DEs are invariants and the boundary conditions will satisfy exactly for all models. In addition we remark that:

$$Q_{\alpha 3} = h(g_\alpha^+ + g_\alpha^-) - 2h \sigma_{\alpha 3}^1, \quad M_{\alpha \beta} = \frac{2h^2}{3} \sigma_{\alpha \beta}^1, \quad \int_{-h}^h t \sigma_{33} dt = h^2 \left(\frac{g_3^+ - g_3^-}{2} - \frac{2}{3} \sigma_{33}^2 \right),$$

$$T_{\alpha \beta} = 2h \sigma_{\alpha \beta}^0, \quad \psi_\alpha = \frac{1}{2} \int_{-h}^h (h^2 - t^2) \sigma_{\alpha 3} dt = \frac{h^2(1 + 2\gamma)}{3} Q_{\alpha 3} + r_2 \left[t \int_0^t \sigma_{\alpha 3} dt; \gamma \right].$$

For the reminder term r_2 [,] see (2.16) of [1]. In this report the realized schemes were created for numerical solution of some BVP of RT for isotropic elastic plates. We used the following methods (see [1], ch.III):

1. Continuous analogue of Peaceman-Rachford alternating direction method,
2. Variational-discrete (projective) methods of approximate solution of some linear 2 Dim boundary value problems for both bounded and unbounded domains [Here, as coordinate functions, are used the spline functions and classical orthogonal polynomials (Legendre, Laguerre, Hermite, ...)],
3. The Alternative to Perturbation Poincaré-Lyapunov's theory, the convergent method for linear operator equation,
4. The parametric derivation method.

This methodology is applied for some practical problems of thin-walled anisotropic elastic structures.

Keywords: Approximate solution, boundary value problems(BVP), thin-walled elastic structures, alternating direction, variational-discrete methods.

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Non-solvability of Balakrishnan-Taylor equation with memory term in $\mathbb{R}^N$

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Abstract

We establish a nonexistence result for a viscoelastic problem with Balakrishnan-Taylor damping and a nonlinear source in the whole space. The nonexistence result is based on the test function method developed by Mitidieri and Pohozaev. We establish some necessary conditions for local existence and global existence as well.

Keywords: Nonexistence, Balakrishnan-Taylor damping, Polynomial kernel.

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Study of third-order three-point boundary value problem with dependence on the first order derivative

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Abstract

The study of boundary value problems for certain linear ordinary differential equations was initiated by Il'in and Moiseev [3] Since then more general boundary value problems for certain nonlinear ordinary differential equations been extensively studied by many authors, see[1, 2, 4].

By using the Leray-Schauder nonlinear alternative, the Schauder contraction theorem and Guo-Krasnosel'skii Theorem we discuss the existence, uniqueness and positivity of solution to an third-order three-point nonhomogeneous boundary value problem

Keywords: nontrivial solution, positive solution, fixed point theoremn, cone.

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On Parameterization and Smoothing of B-splines Interpolating Curves

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Abstract

The behavior of interpolating curves heavily depends on the choice of parameter values associated with interpolating points. We consider plane cubic B-spline curve and do numerical tests on different methods of parameterization [1, 2, 3, 4] in order to show that unwanted effects typically occur when the points are unevenly distributed. We improve parameterization by using an iterative process [5]. We compute first an approximation of the arc length which is the cumulated chord length, noted $cs_i, i = 1; n$, of successive segments $P_{i+1} - P_i, i = 1; n$ then we construct a uniform B-spline curve that interpolate the points $(cs_i, i), i = 1; n$ and choose on this curve specific value as the new parameter for each data point $P_i, i = 1; n$. We propose also a criterion based on dynamics as the termination criterion of the iterative algorithm.

Keywords: Parameterization, interpolation, spline, singularities, fairing

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Statistical extension of some classical Tauberian theorems

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Abstract

Let $s : [1, \infty) \rightarrow \mathbb{C}$ be a locally integrable function (in Lebesgue's sense). We define the harmonic summability and the statistical convergence of the function s at ∞ . We prove the nondiscrete versions of the Landau-type and Hardy-type Tauberian theorems in the case of harmonic summability.

Keywords: statistical limit of functions, harmonic summability of functions, slowly decreasing and slowly oscillating functions

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